$$\text{Southward force} = 2k_1 v_e \cos\theta.$$
$$\text{Eastward force} = -2k_1 v_s \cos\theta.$$
$$\text{Hence,}\quad \text{Total force} = 2k_1 v \cos\theta.$$

The tangent of the direction of the action of this force (from south via east) is

$$-\frac{v_s}{v_e} = \frac{v_s}{v_w},$$

while the tangent of the direction of the moving particle is, of course,

$$-\frac{v_e}{v_n} = \frac{v_e}{v_s}.$$

The force, therefore, acts at right angles to the instantaneous path of the particle, and so is a deflecting force. (*Cf.* Coast and Geodetic Survey Reports, 1900, p. 571; 1904, p. 332.)

STUDIES ON THE VORTICES OF THE ATMOSPHERE OF THE EARTH.

By Prof. FRANK H. BIGELOW. Dated Washington, D. C., March 16, 1908.

IV.—THE DEWITTE TYPHOON, AUGUST 1-3, 1901.

THE METEOROLOGICAL DATA.

In order to illustrate the structure of a hurricane as analyzed by the theory of the dumb-bell-shaped vortex, I have chosen the DeWitte typhoon which occurred in the China Sea August 1-6, 1901. This hurricane is specially valuable for our studies, because the observations at observatories on the coast of China and on the outlying islands afford an unusually large amount of suitably published data. A paper by Rev. Louis Froc, S. J.,[1] and some notes by Rev. José Algué, S. J.,[2] give the isobars, wind directions and velocities at midnight of August 2, 1901. In Table 52 will be found other data extracted from the China Coast Meteorological Register and the Monthly Report of the Central Meteorological Observatory of Japan.

The isobars of August 2, 10 a. m., 10 p. m., August 3, 5 a. m., are reproduced in Chart IX, figs. 9, 10, and 11, respectively. The isobars of August 2, 10 p. m., fig. 10, have been made the basis of the computation, because the typhoon was then at its greatest intensity, the barometer at the center having fallen to about 690 millimeters. Chart IX, fig. 12, shows upon an adopted system of isobars constructed from the vortex data, the wind direction and velocity located according to circumstances within the diagram so as to give a composite view of the vectors on all sides of the axis and at the proper distances from it. The temperatures, fig. 13, and the relative humidity, fig. 14, have been plotted in a similar manner. An inspection of the temperature and relative humidity diagrams, shows that in this case there is no important difference between the western and the eastern quadrants, such as is always found in ordinary cyclones as distinguished from hurricanes. The relative humidity, however, seems to be somewhat higher in the southwest quadrant than in the others, due probably to the excess of the tendency to precipitation in that region. It is very evident that no temperature differences exist in the sea-level horizontal section of the hurricane, such as can account for its energy thru rotations generated by two masses at different temperatures lying side by side on the same level. It is probable that these temperature differences exist in higher levels where a cold sheet overlays a warm sheet, the surface of separation being horizontal rather than vertical.

CONSTRUCTION OF THE AVERAGE HURRICANE VORTEX.

It is our purpose to construct the average vortex which underlies the actual hurricane with all its divergencies due to local conditions. The vortices in the atmosphere are seldom symmetrical, tho the principles of vortex action prevail with more or less perturbation. It is our first study to obtain the symmetrical vortex with all its velocities, angles, and pressures; we can then find the forces which produce the actual vortex, thru a series of differences obtained by subtracting the symmetrical system from the observed data. Thus the progressive northwestward motion of the typhoon makes the wind-angles greater southwest of the center than to the northwest. This angular difference is eliminated as follows: At the northern, eastern, southern, and western points of each isobar construct the appropriate wind vector (the heavy dotted arrows of fig. 12) as accurately as possible from the wind observations taken in the region and plotted on the composite diagram, fig. 12. Then take the mean velocity and the mean angle on each isobar, i. e., the mean values of the four average vectors of each isobar. In the present study the angle 30° has been assumed thruout this horizontal section, whence $i = -30°$ and $az = 60°$, the angular height at which the general vortex is truncated by the sea-level plane, whatever its actual height in meters may be.

TABLE 52.—*Meteorological data*[3] *of the De Witte typhoon, August 1-3, 1901.*

Station.	Latitude. North.	Longitude. East.	Date.	Hour.	*B.*	*t.*	Relative humidity.	Wind.	
	°	°			*Mm.*	°*C.*	*Percent.*	*M.p.s.*	*Dir.*
Gutzlaff	30.49	122.10	Aug. 1	3 p.m.	757.2	28.9	87	7	ese.
			2	9 a.m.	754.9	26.7	80	11	ene.
			2	3 p.m.	......	.....	......	......	.....
			3	9 a.m.	750.8	26.1	95	25	ese.
Sharp Peak	26.7	119.40	Aug. 1	3 p.m.	753.1	28.9	85	11	ne.
			2	9 a.m.	748.3	30.0	86	7	nnw.
			2	3 p.m.	743.5	32.3	91	7	nnw.
			3	9 a.m.	728.5	26.7	78	25	ssw.
Amoy	24.27	118.5	Aug. 1	3 p.m.	753.9	31.6	80	9	se.
			2	9 a.m.	751.1	28.3	83	7	wsw.
			2	3 p.m.	747.5	34.4	94	9	w.
			3	9 a.m.	742.2	30.3	96	16	sw.
Taihoku	25.4	121.28	Aug. 1	10 p.m.	750.4	26.8	91	4	nw.
			2	10 a.m.	742.6	26.2	93	21	nw.
			2	10 p.m.	732.7	24.3	99	27	nw.
			3	5 a.m.	736.8	25.2	86	17	sw.
Taichu	24.2	120.40	Aug. 1	10 p.m.	744.9	25.1	91	4	n.
			2	10 a.m.	738.7	26.1	93	16	nw.
			2	10 p.m	736.3	24.1	98	6	n.
			3	5 a.m.	734.4	26.9	97	15	sw.
Hokoto	23.33	119.34	Aug. 1	10 p.m.	751.7	28.0	85	10	nw.
			2	10 a.m.	747.6	29.5	76	16	nw.
			2	10 p.m.	742.5	28.8	87	12	wsw.
			3	5 a.m.	740.5	28.6	91	17	nw.
Tainau	22.59	120.12	Aug. 1	10 p.m.	750.2	28.1	80	6	nw.
			2	10 a.m.	746.3	28.8	71	12	nw.
			2	10 p.m.	742.7	25.9	100	10	w.
			3	5 a.m.	742.1	28.2	87	17	sw.
Taito	22.45	121.8	Aug. 1	10 p.m.	747.3	27.4	76	4	w.
			2	10 a.m.	739.8	26.0	92	8	sw.
			2	10 p.m.	738.5	27.4	74	21	ssw.
			3	5 a.m.	739.7	27.3	87	17	sw.
Ishigakijima	24.20	124.7	Aug. 1	10 p.m.	740.6	25.8	93	17	n.
			2	10 a.m.	725.5	26.0	100	22	n.
			2	10 p.m.	737.2	27.0	91	37	s.
			3	5 a.m.	744.7	28.7	83	26	s.
Naha	26.13	127.41	Aug. 1	10 p.m.	741.0	25.6	83	21	e.
			2	10 a.m.	742.5	25.5	91	24	ese.
			2	10 p.m.	751.5	26.8	90	10	se.
			3	5 a.m.	752.6	26.5	83	9	se.
Oshima	28.23	129.30	Aug. 1	10 p.m.	753.6	27.9	80	4	e.
			2	10 a.m.	753.7	28.0	76	14	se.
			2	10 p.m.	757.4	27.4	80	10	se.
			3	5 a.m.	758.0	26.6	85	7	ese.

For the mean isobars of the vortex the procedure is as follows: Scale off the distance of the isobars from the center on the north, east, south, and west lines and take the mean of these four as the observed radius, ϖ, of the appropriate vortex tube. Look out the log ϖ of the successive radii and take the successive differences, $\log \rho = \log \varpi_n - \log \varpi_{n+1}$. Finally, take the mean, $\log \rho_m$, and reconstruct the computed $\log \varpi_n$ by

[1] The DeWitte Typhoon, August 1-6, 1901. Annals Zi-ka-wei Observatory.

[2] The Cyclones of the Far East. Manila Observatory. p. 31.

[3] Extracted from the China Coast Meteorological Register and the Monthly Report of Central Meteorological Observatory of Japan.

adding $\log \rho_m$ to the inner radius, which in this typhoon is assumed to be $\varpi_8 = 14{,}000$ meters. On the scale of the diagram, fig. 13, 1° of the map = 96,000 meters. The value of the mean $\log \rho_m$ is smaller for August 2, 10 p. m., than for August 1, 10 a. m., or August 3, 5 a. m., and the diagrams, figs. 9, 10, and 11, show that the isobars are closer on August 2, 10 p. m., than on the preceding or the following dates. The probable pressures, B, radial distances ρ of the isobars in meters and in ϖ_n have been placed on the diagrams, figs. 12, 13, 14, in their northeast quadrants.

In Table 53 will be found the mean measured radii of the several isobars. The inner radii were found by constructing diagrams in two coordinates with B_n and ϖ_n as arguments and drawing a suitable curve to represent both elements. From $\log \varpi$ is computed $\log \rho$ and $\log \rho_m$, and beginning with $\varpi_8 = 14{,}000$ meters the other radii are constructed by adding $\log \rho_m$ in succession. This table also compares the computed ϖ_n with the adopted ϖ_n as derived from the diagrams. In computing the values of u and v, after a few velocities are derived from the observations, the values of v can be extended to the outer and inner tubes without direct velocity readings, since $v\varpi =$ constant, $v = q \cos 30°$, $u = -q \sin 30°$.

TABLE 53.—*The observed and adjusted values of ϖ in the De Witte typhoon.*

B.	Tube.	Measured.		$\log \rho$.	Adjusted.	
		ϖ.	$\log \varpi$.		$\log \varpi$.	ϖ.
Mm.		*Meters.*				*Meters.*
690	ϖ_8	14000	4.14613		4.14613	14000
				0.19629		
700	ϖ_7	22000	4.34242		4.35159	22470
				0.22578		
710	ϖ_6	37000	4.56820		4.55705	36062
				0.23798		
720	ϖ_5	64000	4.80618		4.76251	57878
				0.16230		
730	ϖ_4	93000	4.96848		4.96797	92890
				0.21904		
740	ϖ_3	154000	5.18752		5.17343	149830
				0.19269		
750	ϖ_2	240000	5.38021		5.37889	239272
				0.20412		
760	ϖ_1	384000	5.58433		5.58435	384018
			Mean $\log \rho = 0.20546$			

Table 54 contains the computation of the values of ϖ, u, v, and w on the sea-level plane for $az = 60°$. The only point that needs special consideration is the adopted value of a, the angular constant, as the top of the hurricane is assumed to be in the level 12,000 meters above the sea. Two general reasons lead to this assumption. First, the approach of a hurricane is always heralded by *high cirrus* clouds flying away radially from the center, and in the Tropics this implies an elevation of from 10,000 to 12,000 meters. Second, in the discussion of the hurricane in the International Cloud Report,[4] it was shown that the characteristic disturbance of the atmosphere as evidenced by the high cloud motions, reaches the cirrus with decided strength. It is probable that the upper asymptotic plane of the vortex system is at the 12,000 meters-level, and that the lower asymptotic plane is 6,000 meters below sea-level, so that,

$$a = \frac{180°}{12{,}000 + 6{,}000} = \frac{180°}{18{,}000} = 0.010°.$$

This is the value of the constant adopted for hurricanes and it is one-tenth as large as the corresponding one for the St. Louis tornado. It may be possible to determine these constants, $\log \rho_m$ and a, more accurately in the future and then our computations can be made with greater precision.

COMPUTATION OF ϖ, u, v, w, ON OTHER PLANES.

The values of $\log a\varpi \sin az$, $\log A$, $\log u$, $\log w$ on the 60° plane, as given in Table 54, now follow from the formula and it is only necessary to extend the computations for ϖ, u, v, and w to the other levels, as will be found in Tables 56, 57, 58, and 59. It is necessary to proceed by logarithms thruout for the sake of precision in working with the large numerical values involved. Since some care must be taken to produce $\log Aa\varpi$ correctly in the other levels we give this part of the work in full. By the formula for the radius,

$$\varpi^2 = \frac{\psi}{A \sin az},$$

we first compute ψ from the formula

$$\psi = \frac{v\varpi}{a},$$

and this is easily done since the terms are known. We obtain from Table 54, for tube (1), the constants $\log \psi = \log \frac{v\varpi}{a} = 8.41069$, $\log A = 7.30446$, $\log a = 8.00000 - 10$.

TABLE 54.—*Computation of ϖ, u, v, w, for each radius, ϖ_n, on the sea-level plane, $az = 60°$.*

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Observed u	−3.7	−5.5	−10.0	−15.0	−22.0	−37.0	....	....
Observed v	7.0	11.0	17.0	25.0	41.0	72.0	130.0	....
$\log v$	0.84510	1.04139	1.23045	1.39794	1.61278	1.85733	2.11394	....
$\log \varpi$	5.58435	5.37889	5.17343	4.96797	4.76251	4.55705	4.35159	4.14613
$\log v\varpi$	6.42945	6.42028	6.40388	6.36591	6.37529	6.41438	6.46553	....
mean $\log v\varpi$	$= \log a\psi = 6.41069$							
$\log v$	0.82634	1.03180	1.23726	1.44272	1.64818	1.85364	2.05910	2.26456
v	6.70	10.76	17.27	27.72	44.48	71.39	114.58	183.89

$a = \frac{180}{12000+6000} = 0.010°$ $az = 60°$ $z = 12000.$

$\log a = \log 0.010 = 8.00000$ $\log \sin 60° = 9.93753$, $\log \cos 60° = 9.69897$

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log a\varpi \sin az$	3.52188	3.31642	3.11096	2.90550	2.70004	2.49458	2.28912	2.08366
$\log A$	7.30446	7.71538	8.12630	8.53722	8.94814	9.35906	9.76998	0.18090
A	.002016	.005193	.013375	.034452	.088744	.248922	.588814	1.516690
$\log Aa\varpi$	0.88881	1.09427	1.29973	1.50519	1.71065	1.91611	2.12157	2.32703
$\log u$	−0.58778	−0.79324	−0.99870	−1.20416	−1.40962	−1.61508	−1.82054	−2.02600
u	−3.87	−6.21	−9.97	−16.00	−25.68	−41.22	−66.15	−106.17
$\log w$	7.54302	7.95394	8.36486	8.77578	9.18670	9.59762	0.00854	0.41946
w	.0035	.0090	.0232	.0597	.1537	.3959	1.0199	2.6210

TABLE 55.—*Computation of $\log \varpi$ and $Aa\varpi$ for all levels of tube (1).*

Altitude.	$\log \frac{\psi}{A \sin az}$	$\log \varpi$	$\log Aa\varpi$	$\log \sin az$	$\log \cos az$
°					
$az =$ 90	11.10623	5.55311	0.85757	0.00000	$-\infty$
80	11.11288	5.55644	0.86090	9.99335	9.23967
70	11.13324	5.56662	0.87108	9.97299	9.53405
60	11.16870	5.58485	0.88888	9.93753	9.69897
50	11.22198	5.61099	0.91545	9.88425	9.80807
40	11.29816	5.64908	0.95354	9.80807	9.88425
30	11.40726	5.70363	1.00806	9.69897	9.93753
20	11.57218	5.78609	1.09055	9.53405	9.97299
10	11.86656	5.93328	1.23774	9.23967	9.99335
0	$+\infty$	$+\infty$	$+\infty$	$-\infty$	0.00000

General formulas.

$$a\psi = Aa\varpi^2 \sin az.$$

$$u = -Aa\varpi \cos az.$$

$$v = Aa\varpi \sin az.$$

$$w = 2A \sin az.$$

$$a\psi = v\varpi.$$

$$\tan i = -\cot az = \frac{u}{v}.$$

$$\tan \eta = \frac{w}{v \sec i}.$$

$$q = v \sec i \sec \eta.$$

[4] See Report Chief of Weather Bureau, 1898-99, vol. 2, p. 456.

The application of these general formulas leads to the values in Tables 56, 57, 58, 59, 60, 61, 62, and 63.

TABLE 56.—*Computation of log ϖ and the radius ϖ, for each tube at successive altitudes.*

Values of log ϖ.

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$az =$ 180°	∞	∞	∞	∞	∞	∞	∞	∞
170	5.93328	5.72782	5.52236	5.31690	5.11144	4.90598	4.70052	4.49506
160	5.78609	5.58063	5.37517	5.16971	4.96425	4.75879	4.55333	4.34787
150	5.70363	5.49817	5.29271	5.08725	4.88179	4.67633	4.47087	4.26541
140	5.64908	5.44362	5.23816	5.03270	4.82724	4.62178	4.41632	4.21086
130	5.61099	5.40553	5.20007	4.99461	4.78915	4.58369	4.37823	4.17277
120	5.58435	5.37889	5.17343	4.96797	4.76251	4.55705	4.35159	4.14613
110	5.56662	5.36116	5.15570	4.95024	4.74478	4.53932	4.33386	4.12840
100	5.55644	5.35098	5.14552	4.94006	4.73460	4.52914	4.82368	4.11822
90	5.55311	5.34765	5.14219	4.93673	4.73127	4.52581	4.32035	4.11489
80	5.55644	5.35098	5.14552	4.94006	4.73460	4.52914	4.32368	4.11822
70	5.56662	5.36116	5.15570	4.95024	4.74478	5.53932	4.33886	4.12840
60	5.58435	5.37889	5.17343	4.96797	4.76251	4.55705	4.35159	4.14613

Values of $\varpi = \left(\frac{a\psi}{Aa \sin az}\right)^{\frac{1}{2}}$.

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$az =$ 180°	∞	∞	∞	∞	∞	∞	∞	∞
170	857600	534338	832938	207443	129253	80534	50179	31265
160	611071	380742	237232	147813	92098	57384	35754	22278
150	505389	314900	196205	122250	76172	47460	29571	18425
140	445740	277727	173044	107820	67180	41858	26081	16250
130	408310	254406	158515	98766	61539	38344	23891	14886
120	384018	239272	149083	92890	57878	36062	22469	14000
110	368655	229700	143120	89174	55563	34619	21569	13440
100	360117	224379	139803	87108	54275	33818	21071	13129
90	357367	222663	138735	86444	53860	33559	20910	13028
80	360117	224379	139803	87108	54275	33818	21071	13129
70	368655	229700	143120	89174	55563	34619	21569	13440
60	384018	239272	149083	92890	57878	36062	22470	14000

THE VELOCITIES IN THE DE WITTE TYPHOON.

The lines on fig. 15 of Chart IX show that the maximum of the tangential velocity v is located in the level 3,000 to 4,000 meters above sea-level, which conforms to the facts obtained in the report on the international cloud observations regarding the distribution of the velocity v. The overhang of the upper portion of these lines agrees with the fact that radial outward velocities are first seen in the cirrus levels, 10,000–12,000 meters, and that they appear in advance of the high winds at the surface. Near the upper plane of reference the radial velocity, u, alone survives, and a moderate tendency for the air to flow out from a center over a large sheet will cause violent winds near the axis to supply the resulting losses, according to the vortex laws. If, in the general circulation, a cold sheet of air is brought to overlay a warm sheet, then the pressure difference, which would be discontinuous at the boundary plane, is partly compensated by the movement of the warm air outward in all directions beneath this cold sheet. This movement appears to be a primary cause of the generation of the vortex whose effect is felt at sea-level on the section which truncates it at the elevation $az = 60°$ where the inflowing angle $i = 30°$. The calm core is due to the violent centrifugal force near the axis, and it is here about 1,400 meters, or 9 miles, in diameter. The decreased pressure in this core favors a downpour from the higher levels, of air which warms and tends to clear the sky near the axis. It seems that this truncated vortex conforms to all the broad facts which are observed in connection with hurricanes and typhoons.

TABLE 57.—*Computation of radial velocities u, for each tube and altitude.*

Values of log u.

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$az =$ 180°	∞	∞	∞	∞	∞	∞	∞	∞
170	1.23109	1.43655	1.64201	1.84747	2.05293	2.25839	2.46385	2.66931
160	1.06354	1.26900	1.47446	1.67992	1.88538	2.09084	2.29630	2.50176
150	0.94554	1.15100	1.35646	1.56192	1.76738	1.97284	2.17830	2.38376
140	0.83779	1.04325	1.24871	1.45417	1.65963	1.86509	2.07055	2.27601
130	0.72352	0.92898	1.13444	1.33990	1.54536	1.75082	1.95628	2.16174
120	0.58778	0.79324	0.99870	1.20416	1.40962	1.61508	1.82054	2.02600
110	0.40513	0.61059	0.81605	1.02151	1.22697	1.43243	1.63789	1.84335
100	0 10057	0.30603	0.51149	0.71695	0.92241	1.12787	1.33333	1.53879
90	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞
80	−0.10057	−0.30603	−0.51149	−0.71695	−0.92241	−1.12787	−1.33333	−1.53879
70	−0.40513	−0.61059	−0.81605	−1.02151	−1.22697	−1.43243	−1.63789	−1.84335
60	−0.58778	−0.79324	−0.99870	−1.20416	−1.40962	−1.61508	−1.82054	−2.02600

Values of $u = -Aa\varpi \cos az$.

$az =$ 180°	∞	∞	∞	∞	∞	∞	∞	∞
170	17.03	27.32	43.86	70.38	112.96	181.30	290.97	466.99
160	11.58	18.58	29.82	47.85	76.80	123.27	197.83	317.51
150	8.82	14.16	22.72	36.47	58.53	93.94	150.77	241.97
140	6.88	11.05	17.73	28.46	45.67	73.30	117.64	188.80
130	5.29	8.49	13.63	21.87	35.10	56.34	90.42	145.12
120	3.87	6.21	9.97	16.00	25.68	41.22	66.15	106.17
110	2.54	4.08	6.55	10.51	16.86	27.07	43.44	69.72
100	1.28	2.02	3.25	5.21	8.36	13.42	21.54	34.58
90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
80	−1.28	−2.02	−3.25	−5.21	−8.36	−13.42	−21.54	−34.58
70	−2.54	−4.08	−6.55	−10.51	−16.86	−27.07	−43.44	−69.72
60	−3.87	−6.21	−9.97	−16.00	−25.63	−41.22	−66.15	−106.17

TABLE 58.—*The computation of the tangential velocities for each tube and altitude.*

Values of log v.

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$az =$ 180°	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞
	170	0.47741	0.68287	0.88833	1.09379	1.29925	1.50471	1.71017	1.91563
	160	0.62460	0.83006	1.03552	1.24098	1.44644	1.65190	1.85736	2.06282
	150	0.70698	0.91244	1.11790	1.32336	1.52882	1.73428	1.93974	2.14520
	140	0.76161	0.96707	1.17253	1.37799	1.58345	1.78891	1.99437	2.19983
°	130	0.79970	1.00516	1.21062	1.41608	1.62154	1.82700	2.03246	2.23792
60	120	0.82634	1.03180	1.23726	1.44272	1.64818	1.85364	2.05910	2.26456
70	110	0.84407	1.04953	1.25499	1.46045	1.66591	1.87137	2.07683	2.28229
80	100	0.85425	1.05971	1.26517	1.47063	1.67609	1.88155	2.08701	2.29247
90	90	0.85757	1.06303	1.26849	1.47395	1.67941	1.88487	2.09033	2.29579

Values of the tangential velocity, $v = Aa\varpi \sin az$.

	$az =$ 180°	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	170	3.00	4.82	7.73	12.41	19.92	31.97	51.31	82.34
	160	4.21	6.76	10.85	17.42	27.95	44.86	72.00	115.56
	150	5.09	8.17	13.12	21.06	33.79	54.24	87.04	139.70
	140	5.78	9.27	14.88	23.88	38.32	61.50	98.71	158.43
°	130	6.31	10.12	16.24	26.07	41.84	67.14	107.76	172.95
60	120	6.70	10.76	17.27	27.72	44.48	71.39	114.58	183.89
70	110	6.98	11.21	17.99	28.87	46.34	74.36	119.35	191.55
80	100	7.15	11.47	18.42	29.55	47.43	76.13	122.18	196.10
90	90	7.20	11.56	18.56	29.78	47.80	76.71	123.12	197.60

TABLE 59.—*The computation of the vertical velocities w, for each tube and altitude.*

Values of log w.

Altitude.		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	$az =$ 180°	−∞	−∞	−∞	−∞	−∞	−∞	−∞	−∞
	170	6.84516	7.25608	7.66700	8.07792	8.48884	8.89976	9.31068	9.72160
	160	7.13954	7.55046	7.96138	8.37230	8.78322	9.19414	9.60506	0.01598
	150	7.30446	7.71538	8.12630	8.53722	8.94814	9.35906	9.76998	0.18090
	140	7.41356	7.82448	8.23540	8.64632	9.05724	9.46816	9.87908	0.29000
°	130	7.48974	7.90066	8.31158	8.72250	9.13342	9.54434	9.95526	0.36618
60	120	7.54302	7.95394	8.36486	8.77578	9.18670	9.59762	0.00854	0.41946
70	110	7.57848	7.98940	8.40032	8.81124	9.22216	9.63308	0.04400	0.45492
80	100	7.59884	8.00976	8.42068	8.83160	9.24252	9.65344	0.06436	0.47528
90	90	7.60549	8.01641	8.42733	8.83825	9.24917	9.66009	0.07101	0.48193

Values of the vertical velocity, $w = 2\,A \sin az$.

	$az =$ 180°	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	170	0.0007	0.0018	0.0046	0.0120	0.0308	0.0794	0.2045	0.5267
	160	0.0014	0.0036	0.0091	0.0236	0.0607	0.1564	0.4028	1.0375
	150	0.0020	0.0052	0.0134	0.0345	0.0887	0.2286	0.5888	1.5167
	140	0.0026	0.0067	0.0172	0.0443	0.1141	0.2939	0.7570	1.9499
°	130	0.0031	0.0080	0.0205	0.0528	0.1360	0.3502	0.9021	2.3237
60	120	0.0035	0.0090	0.0232	0.0597	0.1537	0.3959	1.0199	2.6270
70	110	0.0038	0.0098	0.0251	0.0648	0.1668	0.4296	1.1066	2.8505
80	100	0.0040	0.0102	0.0263	0.0679	0.1748	0.4502	1.1597	2.9873
90	90	0.0040	0.0104	0.0268	0.0689	0.1775	0.4572	1.1776	3.0334

THE HORIZONTAL ANGLE i AND VERTICAL ANGLE η OF THE CURRENT q IN THE DE WITTE TYPHOON.

It may be observed by comparison with the Cottage City waterspout and the St. Louis tornado how small the angle η, between the stream line and the horizontal plane, has become in the typhoon. It is only a few minutes in arc, except at the inner tubes (7) and (8), and there it is less than 1°. The helical flow is very flat and the air ascends slowly. In the tornado there is a powerful uplift, but this is lacking in the hurricane, whose destructive winds are nearly horizontal.

TABLE 60.—*The horizontal angle i,*

$$\tan i = \frac{u}{v}.$$

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
°	°	°	°	°	°	°	°	°
az = 180	+90	+90	+90	+90	+90	+90	+90	+90
170	+80	+80	+80	+80	+80	+80	+80	+80
160	+70	+70	+70	+70	+70	+70	+70	+70
150	+60	+60	+60	+60	+60	+60	+60	+60
140	+50	+50	+50	+50	+50	+50	+50	+50
130	+40	+40	+40	+40	+40	+40	+40	+40
120	+30	+30	+30	+30	+30	+30	+30	+30
110	+20	+20	+20	+20	+20	+20	+20	+20
100	+10	+10	+10	+10	+10	+10	+10	+10
90	0	0	0	0	0	0	0	0
80	−10	−10	−10	−10	−10	−10	−10	−10
70	−20	−20	−20	−20	−20	−20	−20	−20
60	−30	−30	−30	−30	−30	−30	−30	−30

TABLE 61.—*The vertical angle η, positive upward,*

$$\tan \eta = \frac{w}{v \sec i}.$$

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
°	' ''	' ''	' ''	' ''	' ''	' ''	' ''	' ''
az = 180	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0
170	0 8	0 13	0 22	0 35	0 55	1 29	2 22	3 49
160	0 23	0 37	0 59	1 36	2 33	4 6	6 35	10 33
150	0 41	1 5	1 45	2 49	4 31	7 15	11 38	18 40
140	1 00	1 36	2 33	4 6	6 35	10 34	16 57	27 12
° 130	1 17	2 4	3 19	5 20	8 34	13 44	22 3	35 23
60 120	1 33	2 29	3 59	6 25	10 17	16 31	29 46	42 32
70 110	1 45	2 49	4 44	7 15	11 38	18 38	29 57	48 4
80 100	1 53	3 1	4 51	7 46	12 29	20 1	32 8	51 34
90 90	1 55	3 5	4 57	7 57	12 46	20 29	32 53	52 46

TABLE 62.—*The total velocity q, im meters per second.*

$$q = v \sec i \sec \eta.$$

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
°								
az = 180	∞	∞	∞	∞	∞	∞	∞	∞
170	17.29	27.75	44.53	71.47	114.71	184.10	295.46	474.20
160	12.32	19.77	31.73	50.93	81.73	131.18	210.53	337.89
150	10.19	16.35	26.24	42.11	67.58	108.47	174.09	279.41
140	8.99	14.42	23.15	37.15	59.62	95.68	153.57	246.47
° 130	8.23	13.21	21.20	34.08	54.61	87.65	140.68	225.78
60 120	7.74	12.42	19.94	32.00	51.86	82.44	132.31	212.36
70 110	7.43	11.93	19.14	30.72	49.31	79.14	127.02	203.86
80 100	7.26	11.65	18.70	30.01	48.17	77.30	124.07	199.15
90 90	7.20	11.56	18.56	29.78	47.80	76.72	123.13	197.62

Volume of air ascending thru each tube of the DeWitte typhoon.

In the Cottage City waterspout the volume of air ascending thru each tube per second was 16,451.5 cubic meters; in the St. Louis tornado it was 774,500 cubic meters; in the DeWitte typhoon it was 1,588,260,000 cubic meters. The typhoon carried 96,540 times as much as the waterspout and 2,050.5 times as much as the tornado, thru each tube. The total volume of air ascending thru all the seven tubes was 11,117,820,000 cubic meters per second. From these values can be computed other interesting quantities.

TABLE 63.—*Logarithms of the volume of air ascending in each vortex tube per second.*

$$V = \pi\,(\varpi_n^2 - \varpi_{n+1}^2)\, w_m.$$

Altitude.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
°								
az = 170	9.20098	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092
150	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092
90	9.20092	9.20092	9.20092	9.20092	9.20092	9.20091	9.20091	9.20091
60	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092	9.20092

EQUATIONS OF MOTION.

If the complete cylindrical equations of motion be written down and the terms substituted as given in the paper on the St. Louis tornado, the partial differentials of the work can be found by multiplying the three equations by $\partial\varpi$, $\varpi\partial\varphi$, and ∂z, respectively. Integrating the equations and adding, also omitting, for the moment, the friction terms, we obtain,

$$(59)\qquad -\frac{P}{\rho} = \tfrac{1}{2}(u^2+v^2+w^2) + \tfrac{1}{2}A^2a^2\varpi^2 + 2A^2\sin^2 az + gz + \text{const.}$$

If now the velocity terms be evaluated they become,

$$(60)\qquad \tfrac{1}{2}\,(u^2+v^2+w^2) = \tfrac{1}{2}\,A^2a^2\varpi^2 + 2A^2\sin^2 az,$$

so that

$$(61)\qquad -\frac{P}{\rho} = A^2a^2\varpi^2 + 4A^2\sin^2 az + gz + \text{a constant.}$$

Integrating between two points and restoring the k-terms, the resulting equation for the work done in transporting the mass whose mean density is ρ_m, becomes,

$$(62)\ -\frac{P}{\rho_m}\Bigg]_n^{n+1} = A^2a^2\varpi^2\Bigg]_n^{n+1} + 4A^2\sin^2 az\Bigg]_n^{n+1} + g(z_{n+1}-z_n) + kq\Bigg]_n^{n+1}.$$

This is the energy required to maintain the circulation under pure vortex conditions, except so far as affected by the coefficient of internal friction. It should be observed that the inertia terms and the expansion or compression terms have each the same value, $\tfrac{1}{2}A^2a^2\varpi^2 + 2A^2\sin^2 az$. It is customary in meteorology to omit the expansion terms, and write the equation of work,

$$(63)\qquad -\frac{P}{\rho} = \tfrac{1}{2}q^2 + gz + \text{a constant;}$$

but in accordance with the above analysis, in the dumb-bell-shaped vortex it is equivalent to

$$(64)\qquad -\frac{P}{\rho} = q^2 + gz + \text{a constant,}$$

Similarly, in the funnel-shaped vortex, we have

$$(65)\qquad -\frac{P}{\rho} = C^2(\varpi^2+4z^2) + gz + \text{a constant,}$$

instead of

$$66)\quad -\frac{P}{\rho} = \tfrac{1}{2}(u^2+w^2) + \tfrac{1}{2}C^2\varpi^2(1-z^2) + 2C^2z^2 + gz + \text{a constant.}$$

The difference in pressure between successive vortex rings.

We will apply the equation for the work of circulation to the DeWitte typhoon on the sea-level section, $az = 60°$ and $i = -30°$, using equation (62). The term in $4A^2\sin^2 az$ will be found very small on the same plane, as it depends only on A^2, and it will be omitted. The values of $\log A^2a^2\varpi^2$ are taken directly from Table 54.

Take the pressure as given for the typhoon on the sea-level plane and apply these differences in succession.

If we take the oblique course of the air from ring to ring in the nearly horizontal helix whose angle from the tangent is 30° inward, then the length of the path is approximately $(\varpi_n - \varpi_{n+1})\sec 30°$. We can obtain the coefficient of friction by using simply the u-velocity and the radial distances. Divide the values of ΔB, the difference between the computed and the observed values of B, by the factor 0.0075 to obtain ΔP, and then divide ΔP by $\rho_m u_m(\varpi_n - \varpi_{n+1})$. We thus find the values of k in Table 64.

The mean coefficient of friction is $k = 0.002740$ for the DeWitte typhoon, while for the St. Louis tornado it was $k = 0.2867$, about 100 times as great. It is quite evident that k is a *variable* coefficient depending upon the conditions prevailing in the section of the vortex under discussion. It may differ from one section to another in the same vortex.

TABLE 64.—*Computation of the difference of pressure ΔB between successive rings.*

Term.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\log A^2 a^2 \varpi^2$	1.77762	2.18854	2.59946	3.01038	3.42130	3.83222	4.24314	4.65406
$A^2 a^2 \varpi^2$	59.93	154.86	397.61	1024.2	2638.2	6795.5	17504.	45088.
$A^2a^2\varpi^2{}_{n+1} - A^2a^2\varpi^2{}_n$		94.43	243.25	626.6	1614.0	4157.3	10708.	27584
		1.97511	2.38605	2.79699	3.20790	3.61881	4.02971	4.44065
$\log \rho_m$		0.06634	0.06054	0.05468	0.04872	0.04270	0.03658	0.03038
log .0075		7.87506	7.87506	7.87506	7.87506	7.87506	7.87506	7.87506
		9.91651	0.32165	0.72673	1.13168	1.53657	1.94135	2.34609
$B_n - B_{n+1}$		0.82	2.10	5.33	13.54	34.40	87.37	221.65
B_o	760	750	740	730	720	710	700	690
B_c	760	759.2	757.1	751.8	738.8	703.9	616.5	395.8
ΔB (in mm.)	0.0	+9.2	+17.1	+21.8	+18.8	−6.1	−83.5	−294.2
FRICTION COEFFICIENTS.								
$\log k$		7.17129	7.44628	7.55761	7.48756	$\times \frac{1}{\sec 60^\circ} = \cos 60^\circ = 0.500.$		
k (friction)		0.001484	0.002794	0.003611	0.003078			

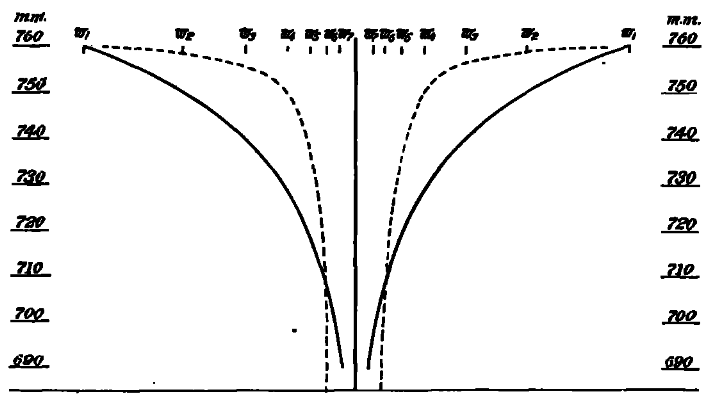

FIG. 16.—The pressures in the DeWitte typhoon, on the sea-level plane.
The - - - - - line shows the pressures without friction.
The ——— line shows the pressures with friction.

The large differences in the pressures as computed by the pure vortex theory and the pressures found at sea level on the weather maps, Chart IX, figs. 9, 10, 11, of the DeWitte typhoon of course need an explanation. There are two sources of divergence between the theory and the observations. The first is the coefficient of internal friction which has been treated as a simple coefficient of the linear velocity. The second is found in the truncation of the great vortex at sea-level, and the consequent cutting off of the natural source of supply for the upper sections of the tube. If these sections are carrying 158,826,000 cubic meters per second, and this should come from the direction of the sea in the tubes which have been truncated and theoretically end in an asymptotic plane which is 6,000 meters below the sea, it follows that this amount of air must be sucked in at or near the surface of the sea from all sides. It is probable that some of the additional centerward pressure gradient in the outer tubes is required to drive the air into the vortex to supply the demands of the upper sections. It is no simple subject to deal with mechanically or mathematically, and further experience must be acquired before it can be successfully considered in greater detail. It may have been better to reduce the values of k given above in the ratio

$$\frac{k}{\sec 60^\circ} = k \cos 60^\circ = 0.5\, k$$

because the path of the wind is oblique to the radius by the angle $az = 60^\circ$ so that the path length is $\sigma = 2.0\, u$ approximately. The pressure difference $B_n - B_{n+1}$ was accounted for in part by the radial friction instead of the friction along the trajectory, but in the St. Louis tornado the lift in the wind, indicated by the angle η, tended to reduce the friction, and it was supposed that the radial path was more likely to give an idea of the value of the coefficient. In the DeWitte typhoon this consideration does not hold true because η is a very small angle.

The trajectories of the wind.

The discussion has proceeded with no regard, heretofore, to the progressive movement of the entire typhoon, which is always a marked feature of hurricanes. The DeWitte typhoon moved due north between 10 a. m. and 10 p. m. August 2, 1901, about 2 degrees at the rate of 16,000 meters per hour, which is 10 miles per hour or 4.4 meters per second. Between 10 p. m., August 2, and 5 a. m., August 3, it moved toward a little north of west, a distance of 3.7 degrees in 7 hours, which is 32 miles per hour or 14.1 meters per second. The causes which produce this translatory movement of the vortical structure are involved in the complex thermodynamic conditions which pertain to the distribution of masses of different temperatures. The laws for this problem have been summarized in my studies on the thermodynamics of the atmosphere, MONTHLY WEATHER REVIEW, Vol. XXXIV, 1906, but the applications of the formulas will require a more extensive knowledge of the temperatures in the upper strata in the neighborhood of the typhoon, than we now possess.

In forming the equations for the trajectories it may be well to make one remark. All trajectories constructed by taking velocities on a circle whose center moves at a given speed are incompetent to discuss these problems fully, for two reasons. Confining the velocity to the tangential component v, and omitting u, w, the planetary equation becomes, when the motion in the circle is equal to the motion of the center,

$$y \sec^2\theta \frac{d\theta}{ds} = 1 - \sec\theta \qquad \text{(Shaw)}, \tag{67}$$

the case for parabolic motion of the particle. If the center moves slower than the particle in the circle, the path becomes an ellipse; if faster, an hyperbola. It is evident, however, that in the pure vortex motion the primary curve of a stationary structure is a logarithmic curve whose equation may be written,

$$r = e^{a\theta}. \tag{68}$$

In a pure vortex the logarithmic spiral changes the angle of inflow, i, from one section to another, so that a series of spirals must be considered. In the Cloud Report, pages 515–519, the formulas for spirals and polar curves generally have been collected, and Table 87 of that report contains the coordinates (r, θ) for different values of the angle a, which corresponds with az in the formulas for the dumb-bell vortex. The trajectory must be formed by adding the motion of the coordinates of the center to those of the moving particle thru the usual differential equations.

The second difficulty in forming the equation for a trajectory is that, aside from tornadoes and the middle group of rings in a hurricane, the vortex law itself begins to break down in the atmosphere. In the outer and in the inner ring of the most perfect portion of the DeWitte typhoon there are evidences of imperfect vortex action. In the ocean and the land cyclones this disintegration proceeds much farther, owing to the different distribution of the thermal masses having different temperatures. In the case of the DeWitte typhoon the trajectories are not built up out of pure logarithmic spirals, but of polar curves only approximating that simple type. It is, therefore, evident that the subject of trajectories, as well as the resistance by friction and internal vortices, must be considered much more fully than it is proper to do in this series of papers.

THE SUN-SPOTS AS HURRICANES OF THE DUMB-BELL VORTEX TYPE.

The sun-spots occur on the outer surface of the photosphere and extend inward toward the center of the sun. They consist visibly of a nucleus which is practically structureless, and a penumbra which is striated radially with much regularity. The observed movements[5] of the material composing the penumbra are from the outer edge of the disturbed area in the photosphere toward the umbra, and the radial striæ usually terminate in ends which are bent downward toward the interior of the sun. The motion of a particle starting on the outer edge of the penumbra is primarily inward and then rather suddenly downward. This corresponds so closely to the motion in the upper levels of a dumb-bell-shaped vortex where the circulation is downward, that it seems proper to suggest this explanation of the origin and structure of the sun-spots. Referring to MONTHLY WEATHER REVIEW, XXXV, October, 1907, p. 475, fig. 3, the sun-spots would correspond to the layers between the sections $az = 180°$ and $az = 170°$, if the circulation is downward. In this limited region there is practically little rotary velocity v, the vertical velocity w becomes important only when approaching the abrupt curvature which is here assumed to be on the outer edge of the umbra, but in the penumbra the radial velocity u is conspicuous. The sun-spot may be caused by layers of matter inside the sun's photosphere operating to draw material downward, warm layers being superposed upon cold layers at the section which corresponds with the lower plane of the sun-spot vortex. There are reasons for considering the sun-spot belts to be cooler areas than those nearer the poles, so that the general circulation would require downward motion from the surface toward the interior. If these views are correct it will become possible to compute the entire vortex system from a few measurements of the radii ϖ and the radial velocity u in the upper layers of the vortex in the region of the surface of the photosphere. If the penumbra is composed of vapors condensed at a certain temperature, their disappearance as visible cloud forms in the hotter layers, as they fall inward and downward, is readily understood. A large series of thermodynamic problems is clearly suggested by this theory, it may properly become the subject of an important research.

[5]A summary of these papers on the vortices in the atmosphere of the earth was read before the National Academy of Sciences at the meeting in Washington, D. C., April 18, 1907.

The photographs of the sun-spot regions secured by Prof. G. E. Hale at the Mount Wilson Solar Observatory in the summer of 1908 are interesting and suggestive in this connection. The curved lines, perhaps paths of motion, there shown probably belong to other levels in the vortex than that herein described, since curvature in the horizontal plane increases with the distance from the asymptote plane. Measures of the angles and velocities should be made with the dumb-bell-shaped vortex in mind.—*F. H. B.*

NOTES ON WEATHER AND CLIMATE MADE DURING A SUMMER TRIP TO BRAZIL, 1908.

By Prof. R. DeC. WARD, Harvard University. Dated Cambridge, Mass., October 15, 1908.

The teacher of climatology should travel. He should, by personal observation, gain some acquaintance with weather types and with climatic conditions in different parts of the world. If he travels equipped with a few portable meteorological instruments and with his eyes open, he will return from each journey to his class-room better equipped as a teacher and better able to interest and instruct his students. The writer has experienced the truth of these assertions very fully in his own case. He hopes that some of his colleagues may be interested in the following more or less haphazard notes which were jotted down at odd intervals during a recent trip to Brazil. This trip was made as a member of the Shaler Memorial Expedition to South America. The writer accompanied the party without official duties, and largely for reasons of his own health. The start from New York was made on June 20, 1908, and Rio de Janeiro was reached July 8. Six weeks were spent in Brazil. On the return voyage the steamer left Rio on August 19, and reached New York September 6. Short stops were made at Bahia and Barbados.

Instrumental equipment.

The instrumental equipment was simple and portable. The following list of instruments is given in the hope that others may find it useful: 2 sling psychrometers; small-size Richard barograph[1]; portable maximum and minimum thermometers[2]; 3-inch rain-gage[2]; nephoscope[3]; Dines's patent portable pressure anemometer[4]; Rotch's instrument for obtaining the true direction and velocity of the wind at sea[5]; a pocket compass. In addition, charts of the North and South Atlantic and a United States Hydrographic Office Pilot Chart of the North Atlantic Ocean for June, 1908, were taken. This equipment proved satisfactory, and as complete as the conditions of ordinary travel warrant.

THE ATLANTIC VOYAGE.

No study of pilot charts or of text-books can give the clear understanding and appreciation of the great wind systems of the world which the traveler who takes an ocean voyage can secure by keeping his eyes open. In June, the month in which the writer started from New York, summer conditions are well established over the North Atlantic. The Pilot Chart shows that the dominant high-pressure area is somewhat to the southwest of the Azores, and covers the central and southern portions of the ocean. From this center, as is well known, the winds blow out spirally.

The prevailing westerlies of the North Atlantic.

To the north of the anticyclone their direction is generally from the southwest, and we have the *prevailing* or *stormy westerlies.* These are often interrupted by cyclones, which cause changes of wind direction to southeast or south with foul weather and rain, followed by a shift to the southwest and west or northwest with clearing weather and higher wind velocities. In these westerlies the pressure changes from day to day are irregular, and often reach 0.50 inch or more. The winds, while generally strong, are variable both in direction and velocity. During the colder months the storms increase in number and are more violent; the shifts of wind are more frequent; the periods of rainy and cloudy weather come oftener and the winds have higher velocities. Because the "Atlantic Ferry" runs thru the latitudes of the stormy westerlies, the passage is apt to take a steamer thru one or more storms, especially in the colder months. There are fewer changes in weather and in pressure on the eastward voyage than on the westward. This is because the storms themselves move eastward, and the

[1] A most interesting traveling companion on an ocean voyage. The barograph was hung from the ceiling of the stateroom by a spiral spring, and was prevented from swinging too violently by means of a string fastened to the side of the room. This same instrument accompanied the writer in 1897-98 on a voyage around South America, and gave a continuous and most interesting record from New York back to New York again.

[2] Not used because of difficulty of proper exposure.

[3] Modified Fornioni pattern (see Cleveland Abbe: Report of Chief Signal Officer for 1887, Part 2, p. 330-331, Pl. XXXII, fig. 86), specially constructed for the writer by Mr. S. P. Fergusson, of Blue Hill Observatory. This nephoscope measures 5½ inches in diameter, and has an adjustable eye-piece in two sections. When used at sea, if the vessel is rolling or pitching, some difficulty is experienced in making an observation, but in smooth seas, such as those met with on the voyage to Brazil, this trouble is reduced to a minimum. For a description of a marine nephoscope mounted on gymbals, see Cleveland Abbe: The Marine Nephoscope. U. S. Weather Bureau Bulletin No. 11, Pt. I, sec. III, p. 161-167, and Pl. VI.

[4] An excellent instrument for use on land. On a moving steamer it is impossible to obtain the true wind velocity directly from anemometer readings. The instrument is made by Casella of London.

[5] A very useful and interesting instrument, described by Prof. A. L. Rotch in the Quart. Journ. Roy. Met. Soc., Vol. XXX, p. 313. The angle between wind direction and the ship's course on these particular voyages to and from Brazil was usually so small that this instrument could not be satisfactorily employed during most of the time.